

American University of Central Asia
Applied Mathematics and Informatics Department
Spring 2016

Syllabus – Partial Differential Equations (PDE) - Equation of Mathematical Physics,
MAT 360, ID 3725

Instructor		Email	Office Hours	Phone	Office
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Course ID	Course Credits	Semester	Day and Time	Room	Lang.
3725	6	Spring	Mo:12.45-14.00 Fr:12.45-14.00	301	English

I. Course Description

Differential equation models describe a wide range of complex problems in biology, engineering, physical sciences, economics and finance. This course focusses on partial differential equation (PDE) models, which will be developed in the context of modelling heat and mass transport and, in particular, wave phenomena, such as sound and water waves. This course develops students' skills in the formulation, solution, understanding and interpretation of PDE models. As well as developing analytic solutions, this course establishes general structures, characterisations, and numerical solutions of PDEs. In particular, computational methods using finite differences are implemented and analysed. Topics covered are: Formulation of PDEs using conservation laws: heat/mass/ wave energy transport; waves on strings and membranes; sound waves; Euler equations and velocity potential for water waves. The structure of solutions to PDEs: separation of variables (space/space, space/time); boundary value problems; Sturm-Liouville theory; method of characteristics; and classification of PDEs via coordinate transformation. Complex-variable form of waves. Wave dispersion. Group velocity. Finite difference solution of PDEs and BVPs: implicit and explicit methods; programming; consistency, stability and convergence; numerical differentiation.

II. Students Learning Objectives:

The two primary goals of many pure and applied scientific disciplines can be summarized as follows:

- i. Formulate/devise a collection of mathematical laws (i.e., equations) that model the phenomena of interest.
- ii. Analyze solutions to these equations in order to extract information and make predictions.

The end result of i) is often a system of partial differential equations (PDEs). Thus, ii) often entails the analysis of a system of PDEs. This course will provide an application-motivated introduction to some fundamental aspects of both i) and ii).

In order to provide a broad overview of PDEs, our introduction to i) will touch upon a diverse array of equations including

- a. The Laplace and Poisson equations of electrostatics;
- b. The diffusion equation, which models e.g. the spreading out of heat energy and chemical diffusion processes;
- c. The Schrödinger equation, which governs the evolution of quantum-mechanical wave functions;
- d. The wave equation, which models e.g. the propagation of sound waves in the linear acoustical approximation;
- e. The Maxwell equations of electrodynamics; and other topics as time permits.

In our introduction to ii), we will study three important classes of PDEs that differ markedly in their quantitative and qualitative properties: elliptic, diffusive, and hyperbolic. In each case, we will discuss some fundamental analytical tools that will allow us to probe the nature of the corresponding solutions.

Student Learning Outcomes

Upon successful completion students should be able to:

1. use knowledge of partial differential equations (PDEs), modelling, the general structure of solutions, and analytic and numerical methods for solutions.
2. formulate physical problems as PDEs using conservation laws.
3. understand analogies between mathematical descriptions of different (wave) phenomena in physics and engineering.
4. classify PDEs, apply analytical methods, and physically interpret the solutions.
5. solve practical PDE problems with finite difference methods, implemented in code, and analyse the consistency, stability and convergence properties of such numerical methods.
6. interpret solutions in a physical context, such as identifying travelling waves, standing waves, and shock waves.

The purpose of this course is to introduce the student not only to the theoretical aspects of differential equations, including the establishment of existence of solutions, but also to techniques for obtaining solutions for the various types of ordinary differential equations.

III. Course Policies

- a. Students are expected to BE ON TIME for classes. If instructor marked the student absent in case that the student is late for the class, he is considered to be absent for the whole class, unless excused by instructor.
- b. ATTENDANCE. Class attendance is required. If the student misses the class with an excuse, he shall provide necessary documents to prove it within a week after he/she missed a class. If the requirements mentioned above are not observed, student's absence is considered to be unexcused.
If a student missed over 15 classes, he/she will not be attested for the course.
- c. WRITTEN ASSIGNMENTS must be submitted to instructor by the deadline. The student may submit assignment late: at the latest by the next day after the deadline before 5 pm, in that case 1 point will be deducted from the final grade for the work (e.g., if your grade is "A" for the work, after deduction, your grade will be "B"). ***This rule applies to any student who was aware or should have been aware of an***

assignment and the deadline no matter whether he was sick or had any other excuse on the date of a deadline.

- d. The student has to follow ACADEMIC HONESTY code. All types of cheating (plagiarism etc) **are strictly prohibited**. If a student fails to observe this requirement, instructor may give from an “F” for the work up to an “F” for the whole course depending on the type of assignment and other circumstances.

IV. Assessment

a. Grading will be based on following components:

Grades will be based on a total of 100 points, coming from:

Quiz 1	The lecturer sets day and time	10 points
Midterm Exam	March, xx, 2016 (The lecturer sets day and time)	30 points
Quiz 2	The lecturer sets day and time	10 points
Final Exam	May, xx, 2016 (The lecturer sets day and time)	40 points
Home works/ Activity	Every class	10 points

b. Grading scale:

The total grade of the student is as follows:

$0 \leq F \leq 40 < D \leq 45 < C- \leq 50 < C \leq 60 < C+ \leq 65 < B- \leq 70 < B \leq 80 < B+ \leq 85 < A- \leq 90 < A \leq 100$

Make-up Exams and Quizzes

- If the reason for missing the midterm exam is valid, the student's final exam will be worth up to 60 points.
- If the reason for missing a quiz is valid, the quiz can be taken at another time and will be worth 5 points.
- If the reason for missing the Final Exam is valid, the student can apply for the grade of “I”.
- If a student misses both exams, he/she will not be attested for the course.
- If the reason for missing any exam or quiz is not valid, then the grade 0 will be given for the missing exam or quiz.

Calculators and cellphones

Using graphic calculators and cell phones during quizzes and exams prohibited.

V. Miscellaneous (as needed or desired)

Prerequisites: Linear Algebra and Analytic Geometry, Mathematical Analysis I, ODE

VI. Textbooks and References

a. Core Text

1. Kreyszig E. Advanced Engineering Mathematics. - John Wiley & Sons, 2006.

2. Strauss A. Walter. Partial Differential Equations. An Introduction. - John Wiley & Sons, 2008.
3. Boyce W., DiPrima R. *Elementary Differential Equations with Boundary Value Problems*. - John Wiley & Sons, 2005.

b. Supplementary Texts

4. Trench W. Elementary Differential Equations with Boundary Value Problems. - Free Edition, 2013.
5. Weiglhofer W., Lindsay K. Ordinary Differential Equations and Applications. - Woodhead Publishing, 2011.

VII. Tentative Academic Calendar

Week 1-4. Wave equation

Basic Concepts. Modeling, Wave Equation, Solution by Separating Variables. Use of Fourier Series, D'Alembert's Solution of the Wave Equation, Characteristics

([1]: ch. 12.1-12.4, [2] : ch. 1, [3] : ch. 1-2, [4] : ch. 1-4, [5] : ch. 1-2)

Week 5-8. Heat Equation

Heat Equation: Solution by Fourier Series, Modeling: Membrane, Two-Dimensional Wave Equation, Rectangular Membrane. Double Fourier Series

Homogeneous Linear ODEs of Second Order, Homogeneous Linear ODEs with Constant Coefficients. Differential Operators. Modeling: Free Oscillations. (Mass-Spring System). Euler-Cauchy Equations. Existence and Uniqueness of Solutions. Wronskian.

Nonhomogeneous ODEs. Modeling: Forced Oscillations. Resonance.: Electric Circuits. Solution by Variation of Parameters.

([1]: ch. 12.5-12.6, [2] : ch. 2, [3] : ch. 3, [4] : ch. 5,6, [5] : ch. 3)

Week 9-10. Laplace equation

Laplacian in Polar Coordinates. Circular Membrane. Fourier-Bessel Series 579 12.10 Laplace Equation in Cylindrical and Spherical Coordinates. Potential

([1]: ch. 12.9-12.11, [2] : ch. 2-3, [3] : ch. 3, [5] : ch. 7)

Week 11-15. Numerical Methods

Methods for First-Order, Multistep Methods, Methods for Systems and Higher Order ODE's, Methods for Elliptic PDEs, Neumann and Mixed Problems. Irregular Boundary, Methods for Parabolic PDEs, Method for Hyperbolic PDEs

([1]: ch. 19.1-21.7, [2] : ch. 4, [3] : ch. 7, [4] : ch. 10, [5] : ch. 8)