American University of Central Asia
Applied Mathematics and Informatics Department
Spring 2016
Syllabus – Functional Analysis, MAT 341, ID 3724

Instructor		**Email**	**Office Hours**	**Phone**	**Office**
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Course ID	**Course Credits**	**Semester**	**Day and Time**	**Room**	**Lang.**
3724	6	Spring	Th:12.45-14.00	205	English

I. Course Description

This course covers major theorems of Functional Analysis that have applications in Harmonic and Fourier Analysis, Ordinary and Partial Differential Equations. This course is a natural follow on of the course Linear Analysis; while the main focus of the Linear Analysis is on Hilbert spaces with its rich geometrical structures this course will work with normed linear spaces. Hilbert space is a special case of a normed linear space. Despite working in this more general framework many results encountered in Linear Algebra will be re-introduced in this course in more general form. For example the spectral theorem will be presented for all compact operators. Review of linear spaces and their norms. The Hahn-Banach, Baire Category, Uniform Boundedness Principle, Open Mapping and Closed Graph theorems. Duality in Banach and Hilbert spaces. Spectral theory for compact operators on Banach spaces. Fredholm alternative.

Students Learning Objectives:

The two primary goals of many pure and applied scientific disciplines can be summarized as follows:

i. Formulate/devise a collection of mathematical laws (i.e., equations) that model the phenomena of interest.
ii. Analyze solutions to these equations in order to extract information and make predictions.

The end result of i) is often a system of partial differential equations (PDEs). Thus, ii) often entails the analysis of a system of PDEs. This course will provide an application-motivated introduction to some fundamental aspects of both i) and ii).

In order to provide a broad overview of PDEs, our introduction to i) will touch upon a diverse array of equations including

a. The Laplace and Poisson equations of electrostatics;
b. The diffusion equation, which models e.g. the spreading out of heat energy and chemical diffusion processes;
c. The Schrödinger equation, which governs the evolution of quantum-mechanical wave functions;
d. The wave equation, which models e.g. the propagation of sound waves in the linear acoustical approximation;
e. The Maxwell equations of electrodynamics; and other topics as time permits.

In our introduction to ii), we will study three important classes of PDEs that differ markedly in their quantitative and qualitative properties: elliptic, diffusive, and hyperbolic. In each case, we will discuss some fundamental analytical tools that will allow us to probe the nature of the corresponding solutions.

Student Learning Outcomes

Upon successful completion students should be able to:

1. Facility with the main, big theorems of functional analysis.
2. Ability to use duality in various contexts and theoretical results from the course in concrete situations.
3. Capacity to work with families of applications appearing in the course, particularly specific calculations needed in the context of Baire Category.
4. Be able to produce examples and counterexamples illustrating the mathematical concepts presented in the course.
5. Understand the statements and proofs of important theorems and be able to explain the key steps in proofs, sometimes with variation.

II. Course Policies

a. Students are expected to BE ON TIME for classes. If instructor marked the student absent in case that the student is late for the class, he is considered to be absent for the whole class, unless excused by instructor.

b. ATTENDANCE. Class attendance is required. If the student misses the class with an excuse, he shall provide necessary documents to prove it within a week after he/she missed a class. If the requirements mentioned above are not observed, student's absence is considered to be unexcused.
If a student missed over 15 classes, he/she will not be attested for the course.

c. WRITTEN ASSIGNMENTS must be submitted to instructor by the deadline. The student may submit assignment late: at the latest by the next day after the deadline before 5 pm, in that case 1 point will be deducted

from the final grade for the work (e.g., if your grade is “A” for the work, after deduction, your grade will be “B”). ***This rule applies to any student who was aware or should have been aware of an assignment and the deadline no matter whether he was sick or had any other excuse on the date of a deadline***.

d. The student has to follow ACADEMIC HONESTY code. All types of cheating (plagiarism etc) **are strictly prohibited**. If a student fails to observe this requirement, instructor may give from an “F” for the work up to an “F” for the whole course depending on the type of assignment and other circumstances.

IV. Assessment

a. Grading will be based on following components:

Grades will be based on a total of 100 points, coming from**:**

Quiz 1	The lecturer sets day and time	10 points
Midterm Exam	March, xx, 2016 (The lecturer sets day and time)	30 points
Quiz 2	The lecturer sets day and time	10 points
Final Exam	May, xx, 2016 (The lecturer sets day and time)	40 points
Home works/ Activity	Every class	10 points

b. Grading scale:

The total grade of the student is as follows:

0 ≤ F ≤ 40 < D ≤ 45 < C- ≤ 50 < C ≤60 < C+ ≤65< B- ≤70 < B ≤ 80 < B+ ≤ 85< A- ≤ 90 < A≤100

Make-up Exams and Quizzes

- If the reason for missing the midterm exam is valid, the student’s final exam will be worth up to 60 points.
- If the reason for missing a quiz is valid, the quiz can be taken at another time and will be worth 5 points.
- If the reason for missing the Final Exam is valid, the student can apply for the grade of “I”.
- If a student misses both exams, he/she will not be attested for the course.
- If the reason for missing any exam or quiz is not valid, then the grade 0 will be given for the missing exam or quiz.

Calculators and cellphones

Using graphic calculators and cell phones during quizzes and exams prohibited.

V. Miscellaneous (as needed or desired)

Prerequisites: Linear Algebra and Analytic Geometry, Mathematical Analysis I, ODE

VI. Textbooks and References

a. Core Text

1. Kreyszig E. Introductory Functional Analysis with Applications - John Wiley & Sons, 1978.
2. Averbuch V. Functional Analysis. - John Wiley & Sons, 1995.
3. Boyce W., DiPrima R. *Elementary Differential Equations with Boundary Value Problems*. - John Wiley & Sons, 2005.

b. Supplementary Texts

4. Trench W. Elementary Differential Equations with Boundary Value Problems. - Free Edition, 2013.
5. Weiglhofer W., Lindsay K. Ordinary Differential Equations and Applications. - Woodhead Publishing, 2011.

VII. Tentative Academic Calendar

Week 1-4. The course will be in three approximately equal parts (so about 4 weeks each).

Normed spaces and a brief treatment of integration
Norms, bounded linear operators, completeness
Step functions, covering lemma, Lebesgue integrable functions
Fatou's lemma, dominated convergence, L1

Week 5-10
Hilbert space
Cauchy's inequality, Bessel's inequality, orthonormal bases
Convex sets, minimization, Riesz' theorem, adjoints
Compact sets, weak convergence, Baire's theorem, uniform boundeness

Week 11-15
Operators on Hilbert space
Finite rank and compact operators
Spectral theorem for compact self-adjoint operators
Fourier series, periodic functions
Dirichlet problem on the interval, completeness of eigenfunctions