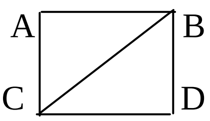
American University of Central Asia
Entrance Examination in Mathematics

Американский университет в Центральной Азии
Вступительный экзамен по математике

№	Problem	Answers (choose the correct one)
1.	Bring the factor into the radical sign: $2\sqrt[5]{b}$ Внесите множитель под знак корня: $2\sqrt[5]{b}$ ***Solution***: $2\sqrt[5]{b}=\sqrt[5]{2^5 b}=\sqrt[5]{32b}$ ***Answer***: $\sqrt[5]{32b}$. (3 points/ 3 балла)	a) $\sqrt[5]{16b^5}$; b) $\sqrt[5]{8b}$; c) $\sqrt[5]{10b^5}$; d) $\sqrt[5]{2^6 b}$; e) $\sqrt[5]{2b^5}$; f) $\sqrt[5]{32b^5}$; g) $\sqrt[5]{32b}$
2.	Calculate the area of a right-angled triangle, if the triangle's legs are $a=30$ and $b=7$. Найти площадь прямоугольного треугольника с катетами $a=30$, $b=7$. ***Solution***: We have a right-angled triangle, and in this case $S=\frac{ab}{2}=\frac{30\cdot 7}{2}=105$ ***Answer: 105.*** (3 points/ 3 балла)	a) 95; b) 105; c) 121; d) 149; e) 151; f) 181; g) 210.
3.	Simplify the expression: $\frac{36-4k^2}{2k-6}$. Упростить выражение: $\frac{36-4k^2}{2k-6}$. ***Solution***: It's easy to rewrite the expression $\frac{36-4k^2}{2k-6}=\frac{-4(k-3)(k+3)}{2(k-3)}=-2(k+3)$ .This answer is equivalent to $-(6+2k)$. ***Answer***: $-(6+2k)$. (3 points/ 3 балла)	a) $6+2k$; b) $-2k+6$; c) $6-2k$; d) $(2k-6)^2$; e) $-(6+2k)$; f) $36+2k$; g) $36-2k$.

4.	The sum of the roots of the equation $3x^2-5x-2=0$ is equal to: Сумма всех корней уравнения $3x^2-5x-2=0$ равна: ***Solution***: Let x_1 *and* x_2 be the roots of equation, then using Viet's Theorem for the equation of second power we can write $x_1+x_2=\frac{5}{3}$, or $\frac{5}{3}=1\frac{2}{3}$. ***Answer***: $1\frac{2}{3}$. (3 points/ 3 балла)	a) $-\frac{3}{5}$; b) $-\frac{1}{2}$; c) $1\frac{2}{3}$; d) $1\frac{3}{4}$; e) $2\frac{1}{3}$; f) $2\frac{2}{3}$; g) 5.
5.	Calculate the area of a square, if the square's diagonal is $2\sqrt{17}$. Найти площадь квадрата, диагональ которого равна $2\sqrt{17}$. ***Solution***: Let's consider a right-angled triangle ABC. Using Pythagorean's theorem, we can write: $\lvert AB\rvert^2+\lvert BC\rvert^2=\left(2\sqrt{17}\right)^2$ in a square $\lvert AB\rvert=\lvert BC\rvert$. $\Rightarrow 2\lvert AB\rvert^2=17\cdot 4\Rightarrow \lvert AB\rvert^2=34.$ In this case, the area of the square is $S=\lvert AB\rvert^2=34$.  ***Answer***: ***34***. (3 points/ 3 балла)	a) 14; b) 28; c) 30; d) 34; e) 49; f) 51; g) 63.
6.	Calculate a number, if it is known that 5% of this number is equal to 23% of 15.5. Найти число, если 5 % его составляют 23% от 15,5. ***Solution***: Let ***a*** be the number we are trying to find. Then it is easy to write equation: $0{,}05a=15{,}5\cdot 0{,}23\Rightarrow a=71{,}3$. ***Answer: 73,1.*** (4 points/ 4 балла)	a) 28,2; b) 33,2; c) 35,1; d) 38,1; e) 41,3; f) 68,2; g) 71,3..

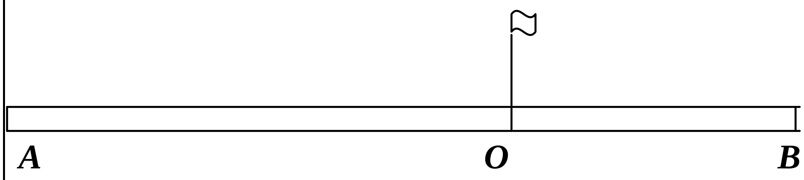
7.	Calculate: $\left(\left(\frac{2}{3}\right)^{-1}-\left(\frac{4}{3}\right)^{-1}\right)^{-1}\cdot 3$ Вычислить: $\left(\left(\frac{2}{3}\right)^{-1}-\left(\frac{4}{3}\right)^{-1}\right)^{-1}\cdot 3$ ***Solution:*** Let's take into account that: $\left(\frac{2}{3}\right)^{-1}=\frac{3}{2}$ *and* $\left(\frac{4}{3}\right)^{-1}=\frac{3}{4}$ Then $\left(\frac{3}{2}-\frac{3}{4}\right)^{-1}\cdot 3=\frac{4}{3}\cdot 3=4.$ ***Answer: 4.*** (4 points/ 4 балла)	a) -4; b) -1; c) 0; d) 3; e) 4; f) 6; g) 7.
8.	Calculate: $\log_2 80-\log_2 5$ Вычислить: $\log_2 80-\log_2 5$ ***Solution***: Using properties of the function $\log_a x$, we can rewrite the expression as: $\log_2 80-\log_2 5=\log_2(2^4\cdot 5)-\log_2 5=$ $=\log_2(2^4)+\log_2 5-\log_2 5=4$ ***Answer***: ***4***. (4 points/ 4 балла)	a) $\log_2 75$; b) 16; c) 4; d) 3; e) 2; f) 0; g) -16.
9.	The sum of all roots of the equation $x^2-\lvert x\rvert-6=0$ is equal to: Сумма всех корней уравнения $x^2-\lvert x\rvert-6=0$ равна: ***Solution:*** We must consider two cases. So: $\left[\begin{array}{l}\begin{cases}x\ge 0\\ x^2-x-6=0\end{cases}\Rightarrow\begin{cases}x\ge 0\\ x_1=3;\ x_2=-2\end{cases}\\ \begin{cases}x<0\\ x^2+x-6=0\end{cases}\Rightarrow\begin{cases}x<0\\ x_1=-3;\ x_2=2\end{cases}\end{array}\right.$ But $x_2=-2$ does not satisfy the condition $x\ge 0$, and	a) -3; b) -2; c) -1; d) 0; e) 1; f) 2; g) 6.

	$x_2 = 2$ does not satisfy the condition $x < 0$. So in this case we only have two roots $x_1 = 3$ *and* $x_2 = -3$. The sum of all the roots of the equation is equal to 0. ***Answer: 0.*** (4 points/ 4 балла)	
10.	The range of admissible argument's values of the function $f(x) = \frac{1}{\sqrt{x+7}} + \frac{1}{(x-3)^2}$ is equal to: Область допустимых значений аргумента функции $f(x) = \frac{1}{\sqrt{x+7}} + \frac{1}{(x-3)^2}$ равна: ***Solution***: The square root exists if $x + 7 \geq 0$, and the fraction exists when denominator is not equal to zero: If we take into account that : $\begin{cases} x+7 \geq 0 \\ x \neq -7 \\ x \neq 3 \end{cases}$, we have the interval: (-7,3) $\cup (3, \infty)$. ***Answer***: (-7,3) $\cup (3, \infty)$. (4 points/ 4 балла)	a) (-7,3) $\cup (3,\infty)$; b) (-7,-3) $\cup (3,\infty)$; c) (-7,0) $\cup (0,3)$; d) (-7,-3) $\cup (3,7)$; e) (-3,0) $\cup (0,\infty)$; f) (-3,0) $\cup (0,3)$; g) (-3,3) $\cup (3,7)$.
11.	If $\sin\alpha + \cos\alpha = \kappa$, then the expression $\sin\alpha \cdot \cos\alpha$ equals : Если $\sin\alpha + \cos\alpha = \kappa$, то выражение $\sin\alpha \cdot \cos\alpha$ равно: ***Solution:*** Let's raise each term on term both sides of the equation to the second power: $(\sin\alpha + \cos\alpha)^2 = (k)^2 \Rightarrow \sin^2\alpha + 2\sin\alpha \cdot \cos\alpha + \cos^2\alpha$. Using the property $\sin^2\alpha + \cos^2\alpha = 1$, we can rewrite $2\sin\alpha \cdot \cos\alpha = k^2 - 1 \Rightarrow \sin\alpha \cdot \cos\alpha = \frac{k^2-1}{2}$. ***Answer***: $\frac{k^2-1}{2}$. (6 points/ 6 баллов)	a) $\frac{\kappa^2}{2}$; b) $\frac{1-\kappa^2}{4}$; c) $\frac{2+\kappa^2}{2}$; d) $\frac{\kappa^2-4}{2}$ e) $\frac{\kappa^2-1}{2}$; f) $\frac{2\kappa^2}{5}$; g) $\frac{\kappa^2+1}{2}$.

12.	The least integer solution of the inequality $\frac{x^2(x-2)}{(x-7)} \le 0$ is equal to: Наименьшее целое решение неравенства $\frac{x^2(x-2)}{(x-7)} \le 0$ равно: ***Solution***: Because $x^2 \ge 0$, we have to satisfy the conditions: $\frac{(x-2)}{(x-7)} \le 0 \ and \ x=0 \Rightarrow x \in [2;7) \ and \ x=0$. The least integer solution of this inequality is 0. ***Answer: 0.*** (6 points/ 6 баллов)	a) -7; b) -2; c) 0; d) 2; e) 3; f) 7; g) 10.
13.	The second term of an arithmetic progression is 0.5 and the third is 0.7. Calculate the sum of the first eight terms of this progression. Второй член арифметической прогрессии равен 0,5, третий ее член равен 0,7. Найти сумму первых восьми членов этой прогрессии. ***Solution****:* We can write system of equations: $\begin{cases} a_2 = 0,5 \\ a_3 = 0,7 \end{cases} \Rightarrow d = a_3 - a_2 = 0,2$ Taking into account $a_n = a_1 + (n-1)d$, we can find a_1 : $a_2 = a_1 + d \Rightarrow a_1 = 0,5 - 0,2 = 0,3$. Then , we have: $\begin{cases} d = 0,2 \\ a_1 = 0,3 \end{cases}$ We can find the sum of the first eight terms of this progression using the formula: $S_n = \frac{2a_1 + (n-1)d}{2} \cdot n \Rightarrow S_8 = \frac{2(0,3) + 0,2 \cdot 7}{2} \cdot 8 = 8$ ***Answer**:**8**.* (6 points/ 6 баллов)	a) 4; b) 5; c) 6; d) 7; e) 8; f) 9; g) 10.

14.	The least integer solution of the inequality $-1 < 2^{X-1} - 5 < 11$ equals: Наименьшее целое решение неравенства $-1 < 2^{X-1} - 5 < 11$ равно: ***Solution***: The equivalent system of inequalities is: $\begin{cases} 2^{X-1} > 2^2 \\ 2^{X-1} < 2^4 \end{cases} \Rightarrow \begin{cases} x-1 > 2 \\ x-1 < 4 \end{cases} \Rightarrow \begin{cases} x > 3 \\ x < 5 \end{cases} \Rightarrow x = 4.$ ***Answer: 4.*** (6 points/ 6 баллов)	a) 2; b) 3; c) 4;; d) 5; e) 6; f) 7; g) 8.
15.	Solve the system of equations $\begin{cases} x(y+2)+2y=16, \\ x(3y-2)-2y=8, \end{cases}$ and calculate the maximum value of $x-y$. Решить систему уравнений $\begin{cases} x(y+2)+2y=16, \\ x(3y-2)-2y=8, \end{cases}$ и найти наибольшее значение величины $x-y$. ***Solution***: We can rewrite the system of equations in the form: $\begin{cases} xy+2x+2y=16, \\ 3xy-2x-2y=8, \end{cases}$ The sum of (1) and (2) equations will give: $4xy = 24 \Rightarrow xy = 6$. Multiply the first equation by 3 and calculating difference between these equations, we will have: $8(x+y) = 40 \Rightarrow x+y=5$. So, we have system of equations: $\begin{cases} x+y=5 \\ xy=6 \end{cases} \Rightarrow \begin{cases} x=5-y \\ (5-y)y=6 \end{cases} \Rightarrow \begin{cases} x=5-y \\ y^2-5y+6=0 \end{cases}$ It's easy to find that there are two pair of roots: $\begin{cases} y=3 \\ x=2 \end{cases}$ *and* $\begin{cases} y=2 \\ x=3 \end{cases}$. Thus, the maximum value of	a) -2; b) -1; c) 0; d) 1; e) 2; f) 4; g) 5.

	$x-y$ is equal to 1. ***Answer: 1.*** (6 points/ 6 баллов)	
16.	The first tap fills the tank with water four times faster than the second tap. If the first half of the tank is filled by the first tap and the second half by the second tap, then it takes 9 hours to fill the tank. How many hours will it take the first tap alone to fill the tank? Первый кран заполняет емкость в четыре раза быстрее, чем второй кран. Если первую половину емкости заполнит первый кран, а вторую половину второй кран, то на это уйдет 9 часов. За сколько часов заполняет емкость один первый кран? ***Solution***: Let productivity of second tap equal x. The productivity of the first tap is 4x, and the tank or job is equal to 1. We need to find the result of expression $\frac{1}{4x}$. Now we have equation: $\frac{1}{2\cdot 4x}+\frac{1}{2x}=9 \Rightarrow 4x=\frac{5}{18} \Rightarrow \frac{1}{4x}=3\frac{3}{5}$. ***Answer***: $3\frac{3}{5}$. (7 points/ 7баллов)	a) $2\frac{1}{2}$; b) $3\frac{3}{5}$; c) $4\frac{1}{3}$; d) $4\frac{2}{3}$; e) $5\frac{1}{2}$; f) $5\frac{3}{5}$; g) $6\frac{3}{5}$.
17.	The road from Almaty to Bishkek takes 5 hours by taxi and 7 hours by bus. A taxi and a bus departed simultaneously with constant speeds toward each other from Almaty and Bishkek respectively. After how many hours will they meet? Путь из Алматы в Бишкек такси проезжает за 5 часов, а автобус за 7 часов. Если они выедут одновременно друг другу навстречу с постоянными скоростями, то через сколько часов такси и автобус встретятся? ***Solution***: Let the speed of the taxi be ***x km/h***, and the speed of the bus be ***y km/h*** . It is obvious that from one side $\lvert AB\rvert = 5x\,km$, and also $\lvert AB\rvert = 7y\,km$.So, $x=\frac{7}{5}y$.	a) $2\frac{1}{12}$; b) $2\frac{1}{6}$; c) $2\frac{11}{12}$; d) $3\frac{5}{12}$; e) $3\frac{7}{12}$; f) $3\frac{11}{12}$; g) $4\frac{5}{12}$.

	Assume they meet at stage **O**. $\lvert AO\rvert+\lvert OB\rvert=\lvert AB\rvert=5x\ or\ 7y$. Let t be hours after they departed until they meet, then  $tx+ty=7y$. Using $x=\frac{7}{5}y$, it is easy to see that $$t=\frac{7y}{\frac{7}{5}y+y}\Rightarrow t=\frac{35y}{12y}=2\frac{11}{12}\ .$$ ***Answer***: $2\frac{11}{12}$. (7 points/ 7баллов)	
18.	The sum of all roots of the equation $\lvert x+1\rvert^{x^2-x-2}=(2012)^0$ equals to: Сумма всех корней уравнения $\lvert x+1\rvert^{x^2-x-2}=(2012)^0$ равна: ***Solution***: It is easy to see that our equation exists when $x\neq-1$. We must consider three cases: $\left[\begin{array}{l} x+1=1;\\ x+1=-1;\\ x^2-x-2=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=0;\\ x=-2;\\ x_1=2;\ \ x_2=-1.\end{array}\right.$ But $x\neq-1$ and so the sum of all roots of our equation is equal to ***0.*** ***Answer***: ***0***. (7 points/ 7баллов)	a) -2; b) -1; c) 0; d) 1; e) 2; f) 3; g) 4.

19.	The least integer solution of the inequality $\log_{2x-1}13 > \log_{2x-1}3$ equals: Наименьшее целое решение неравенства $\log_{2x-1}13 > \log_{2x-1}3$ равно: ***Solution***: The domain of functions $\log_{2x-1}13$ *and* $\log_{2x-1}3$ means: $\begin{cases} 2x-1 \neq 1 \\ 2x-1 > 0 \end{cases} \Rightarrow \begin{cases} x \neq 1 \\ x > \frac{1}{2} \end{cases}$. Taking into account that 13>3, we require: $2x-1 > 1 \Rightarrow x > 1$. Consequently, we have this system of the inequalities: $\begin{cases} x \neq 1 \\ x > \frac{1}{2} \\ x > 1 \end{cases} \Rightarrow x > 1$. The least integer solution of the inequality is 2. ***Answer***: ***2***. (7 points/ 7баллов)	a) 1; b) 2; c) 3; d) 4; e) 5; f) 6; g) 7.

20.	The sum of all roots of the equation $\lvert x+2\rvert-\lvert x-3\rvert+\lvert x-1\rvert=4$ equals: Сумма всех корней уравнения $\lvert x+2\rvert-\lvert x-3\rvert+\lvert x-1\rvert=4$ равна: ***Solution***: We need to consider four cases: $\begin{cases} x<-2 \\ -(x+2)+(x-3)-(x-1)=4 \end{cases} \Rightarrow x=-8;$ $\begin{cases} -2\le x<1 \\ (x+2)+(x-3)-(x-1)=4 \end{cases} \Rightarrow \quad x\in\varnothing;$ $\begin{cases} 1\le x<3 \\ (x+2)+(x-3)+(x-1)=4 \end{cases} \Rightarrow \quad x=2;$ $\begin{cases} x\ge 3 \\ (x+2)-(x-3)+(x-1)=4 \end{cases} \Rightarrow x\in\varnothing.$ The sum of all roots of the equation is (-6). ***Answer*: *(-6).*** (7 points/ 7баллов)	a) -6; b) -4; c) -2; d) 0; e) 2; f) 4; g) 6.